Mathematical Modeling of Nonholonomic Vehicle's Autonomous Navigation Using Fuzzy Logic

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Abstract— **This work describes the mathematical modeling of a navigation strategy for a non- holonomic car like robotic vehicle to a target position. Only the starting and end points are specified, and the robot itself works out the trajectory. Trajectory-traversing algorithm has been named as Linear Paths Approximation (LPA) method as the technique is based on estimating small linear segments to traverse the path between source and target position. Design of a trajectory traversing technique using LPA has been integrated with fuzzy logic based steering control system. Proposed system has been successfully tested for path tracing in simulation environment using MATLAB.**

Keywords: Autonomous Navigation, Mathematical Modeling, Path Tracing, Robotics, Nonholonomic Vehicles.

I. Introduction

Autonomous navigation can be defined as purposeful navigation of a mobile robot without human intervention to achieve a target position with the required orientation. Autonomous navigation is a benchmark research area because of its vast majority of applications, be it in industrial automation, social or civil, precision agriculture or space exploration works. The type of the robotic vehicle, it's physical & kinematics parameters are decided as per the requirements of the application area. The different constraints on their motion together with the physical constraints like size and maximum speed are known as the Robot's Kinematics Constraints. Mobile robots come in two categories as far as the type of the motion is concerned which are holonomic and nonholonomic.

A. Holonomic Vehicles

Some of commercial and research robots are holonomic. Such vehicles have decoupled translation and rotation motion. Holonomic robots are able to vary each component of their position and orientation independently. By turning on the spot they can move in any direction regardless of their orientation. Therefore they can easily reach the final orientation-angle by means of a single rotation, once the 2D position has been attained. It is very difficult to design a true holonomic vehicle as there are always some limitations imposed by the physical design of the vehicle on its kinematics. The design of the holonomic vehicles requires the intensive study of the carriage unit, the specialized design of the wheels, their rotational & control mechanism.

B. Nonholonomic Vehicles

They may not be able to change their orientation without changing position and/or may only be able to move in limited number of directions depending on their shape & orientation of wheels/legs. Therefore Nonholonomic vehicles are forced to translate their motion for turning to a final orientation. Thus nonholonomic vehicles require a much finer and intelligent control strategy than holonomic ones. Examples of nonholonomic four wheelers are robots such as ROJO, which has been successfully used in area of agriculture, others are four wheeler commercial vehicles such as cars, trucks etc.

Auto navigation is also applicable to many hazardous or tedious navigation tasks which can be both more efficiently accomplished and more environmental & human respectful, such as in pesticide spraying or de-mining operations etc. Driving and parking of nonholonomic vehicles require a high degree of expertise, as non-trivial and often complex maneuvering are needed to reach a close location with a final orientation when it is highly deviated from the initial one. Therefore it is important that a study should be conducted on the navigation techniques of nonholonomic robots.

II. Literature survey

Scheuer et al [1] in 1996, presented continuous-curvature path planner (CCPP), one of the first planners for car-like robots which could compute the collision

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free path consisting of straight segments connected with tangential circular arcs. The discontinuities constrained the motion in respect that car would have to stop at these points and reorient its front wheels in the desired direction.

Scheuer et al [2] in 1997 extended their earlier work [1] to remove the motion constraint at discontinuities by designing a path comprising of pieces, each piece being a line segment, a circular arc of maximum curvature or a clothoid arc, called as simple continuous curvature paths(SCC). The earlier limitation was removed by adding the continuous curvature constraint and a constraint on the curvature derivative to imply that car like vehicle could orient its front wheels with a small finite velocity. Result was a first path planner for a car like vehicle to generate collision free path with continuous curvature and maximum curvature derivative which was experimentally verified.

Igor E. Paromtchik [6], in 2004, addressed planning control commands of the steering angle and velocity for low-speed autonomous parking maneuvers in a constrained traffic environment, by making use of conformity between the control commands and resulting shape of the path. The path shape required for a parking maneuver was evaluated from the environmental model. The corresponding control commands were selected and parameterized to provide motion within the available space, to be executed by the car servo systems to drive the vehicle into the parking place. The approach was tested on a CyCab automated vehicle. The experimental results on a perpendicular parking maneuver were described; the obtained results proved the effectiveness of autonomous perpendicular and parallel parking maneuvers.

M.Yousef Ibrahim et al [7], in 2004, discussed on various autonomous navigation techniques. They discussed different Navigation methodologies such as Simultaneous Localization and Mapping (SLAM) which involves clubbing of two techniques, Localization techniques i.e. the problem of identifying the robot's position in the environment with respect to known features, & Map building techniques i.e. to construct an internal model of any unknown features in the environment. Line segment techniques which extract line segments from adjacent collinear range measurements to develop a line segment map & then using a fast matching algorithm to compare this local line-segment map to a global map, the robot can reduce any position error developed from slippage and drift. Another novice technique Artificial Potential Field based which involves modeling the robot as a particle in space, acted on by some combination of attractive and repulsive fields. In this technique obstacles and goals are modeled as charged surfaces, and the net potential generates a force on the robot. These forces push the robot away from the obstacles, while pulling it towards the goal. The robot moves in the direction of greatest negative gradient in the potential. Despite its simplicity this method can be effectively used in many simple environments.

Nascimento TP et al [16] proposed approach in trajectory tracking by using nonlinear model predictive controller to track a given trajectory. Modifications were introduced in the robot model, cost function, and optimizer aiming to minimize the steady-state error rapidly. Authors compared results of simulations and experiments with real robots.

Zhang J et al [17] proposed autonomous parallel parking decoupled into the path planning problem and the longitudinal velocity planning problem to reduce the difficulty of the trajectory planning problem. A collision-free path is designed by combining circle arcs with straight line, and then the path is transformed into a continuous-curvature path using B-spline curve, longitudinal velocity is created using B-spline curve, finally, the parking performance is verified based on model-in-the-loop simulation system.

Roy S et al [18] presented a robust hybrid control method for efficient path tracking under parametric and nonparametric variations, aimed at reducing the effort required for modeling the complex wheeled mobile robotic systems by approximating the unknown dynamics using input and feedback information of past time instances.

Chae HW et al. [19] proposed autonomous navigational algorithm named keyframe-based autonomous visual-inertial navigation (KAVIN) using only a stereo inertial sensor without relying on wheel-based dead reckoning. Matveev AS et al. [20] presented a novel computationally efficient navigation method in simulation environment with fairly regular behavior to drive the nonholonomic robot through the obstacles-free path of the plane to the curve (isoline), without estimating of the field gradient. Muñoz-Vázquez et al in [21] presented a novel contour tracking scheme for nonholonomic mobile robots using fuzzy aggregation of spatial sets and obstacle avoidance was achieved by combining spatially distributed velocity fields.

III. Proposed Method of Trajectory Traversing

Proposed method of trajectory traversing has been named as Linear Paths Approximation (LPA). In this method, motion of the vehicle is divided into two parts: - main component at front axle and its effect in turn at rear axle. For this two geometrical points are considered:

(xa, ya)	:	Front axle mid point
(x, y)	:	Rear axle mid point

Both the points are subjected to a small linear motion. Let point *(xa, ya)* is subjected to a small linear motion in the direction parallel to the direction of front wheels. As shown in the Figure 1, points (x, y) & (xa, ya) are subject to transformation (x', y') & (xa', ya') respectively. This transformation in θ_T is taken as input to our trajectory calculation algorithm to get the outcome transformations as shown in the block diagram representation in Figure 1.

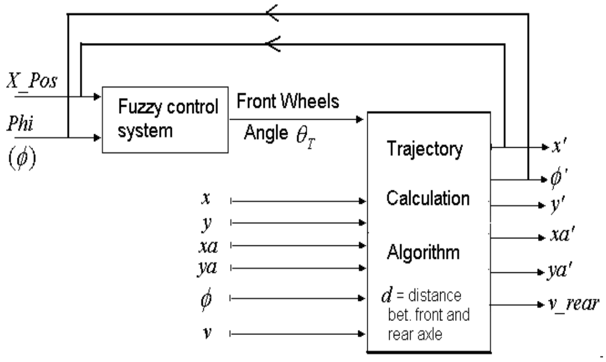

Fig. 1: Linear Paths Approximation (LPA) Based System Block Diagram

Where

v : Vehicle's velocity being exhibited by front axle mid point.

v_rear : Resultant velocity exhibited by rear axle mid point.

As per Figure 2, following transformations have to be calculated to completely define the new position of the vehicle:

$(x, y, \phi) \rightarrow (x', y', \phi')$

$(xa, ya) \rightarrow (xa', ya')$

$v \rightarrow v\_rear$

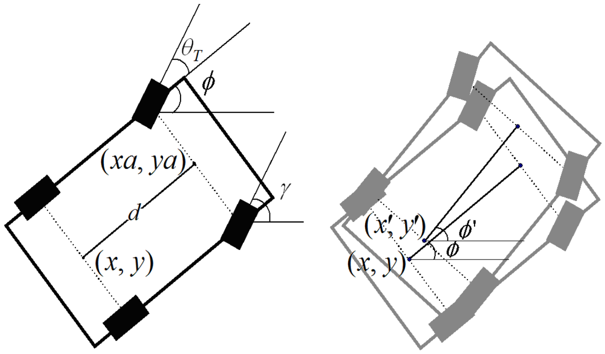

Fig 2: Linear Paths Approximation Method for Trajectory Tracing

As per Figure 2,

$gamma = \gamma = \theta_T + \phi$, where

γ : Angle of the front wheels with respect to horizontal axis. Therefore it is also subjected to transformation as:

$\gamma \rightarrow \gamma'$

Let us say a small linear motion Δd is applied at (xa, ya). If we have to perform every iteration after time interval Δt ,

$\Delta d = v \times \Delta t$

For the unit time interval,

$\Delta d = v$

So transformation of (xa, ya) can be formulated as:

$$xa' = xa + v.\sin(\gamma') \quad (1)$$

$$ya' = ya + v.\sin(\gamma') \quad (2)$$

From Figure 2, transformation of (x, y) can be formulated as:

$$x' = x + v\_rear.\cos(\phi') \quad (3)$$

$$y' = y + v\_rear.\sin(\phi') \quad (4)$$

As per the geometry of the vehicle, points (x, y) & (xa, ya) can also be calculated as:

$$xa = x + d.\cos(\phi) \quad (5)$$

$$ya = y + d.\sin(\phi) \quad (6)$$

$$xa' = x' + d.\cos(\phi') \quad (7)$$

$$ya' = y' + d.\sin(\phi') \quad (8)$$

From equations (1, 5 & 7)

$xa' = xa + v.\cos(\gamma')$ i.e.

$x' + d.\cos(\phi') = [x + d.\cos(\phi)] + v.\cos(\gamma')$

From equation (3)

$[\{x + v\_rear.\cos(\phi')\} + d.\cos(\phi')]$

$= [x + d.\cos(\phi)] + v.\cos(\gamma')$

$$(v\_rear + d)\cos(\phi') = d.\cos(\phi) + v.\cos(\gamma') \quad (9)$$

Similarly from equations (2, 4, 6 & 8)

$$(v\_rear + d)\sin(\phi') = d.\sin(\phi) + v.\sin(\gamma') \quad (10)$$

Dividing equation (3.9) by (3.10) and rearranging we get

$$d.[\sin(\phi')\cos(\phi) - \cos(\phi')\sin(\phi)] = v.[\sin(\gamma')\cos(\phi') - \cos(\gamma')\sin(\phi')] \quad (11)$$

=> $d.[\sin(\phi' - \phi)] = v.[\sin(\gamma' - \phi')]$

Where $\phi' - \phi$ = Vehicle's Steer = Steer_V

and $\gamma' - \phi' = \theta_T'$

So equation can be re-written as:

$d[\sin(Steer\_V)] = v[\sin(\theta_T')]$

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$$=> Steer\_V = \sin^{-1}\langle \frac{v}{d}.\sin(\theta_T')\rangle$$

$$\phi' = \phi + \sin^{-1}\langle \frac{v}{d}.\sin(\theta_T')\rangle \quad (12)$$

From equation (10)

$$(v\_rear + d)\ \sin(\phi') = d.\sin(\phi) + v.\sin(\gamma')$$

$$v\_rear = \frac{d.\sin(\phi) + v.\sin(\gamma') - d.\sin(\phi')}{\sin(\phi')} \quad (13)$$

$$v\_rear.\sin(\phi') = d.\sin(\phi) + v.\sin(\gamma') - d.\sin(\phi') \quad (14)$$

From equation (9)

$$(v\_rear + d)\ \cos(\phi') = d.\cos(\phi) + v.\cos(\gamma')$$

i.e.

$$v\_rear.\cos(\phi') = d.\cos(\phi) + v.\cos(\gamma') - d.\cos(\phi') \quad (15)$$

From (3), (4), (14) and (15)

$$x' = x + d.\cos(\phi) + v.\cos(\gamma') - d.\cos(\phi') \quad (16)$$

$$y' = y + d.\sin(\phi) + v.\sin(\gamma') - d.\sin(\phi') \quad (17)$$

Final kinematics equations have been presented in Table 1.

TABLE 1: KINEMATICS EQUATIONS DESCRIBED BY LINEAR PATHS APPROXIMATION ALGORITHM.

Transformation	Equations
$\phi \dashrightarrow \phi'$	$\phi' = \phi + \sin^{-1}\langle \frac{v}{d}.\sin(\theta_T')\rangle$
$x \dashrightarrow x'$	$x' = x + d.\cos(\phi) + v.\cos(\gamma') - d.\cos(\phi')$
$y \dashrightarrow y'$	$y' = y + d.\sin(\phi) + v.\sin(\gamma') - d.\sin(\phi')$

IV. RESULTS & DISCUSSION

The proposed LPA algorithm has been tested in simulation environment in MATLAB. Figure 3 shows the results of path tracing by LPA algorithm in simulation environment. Three simulations have been performed corresponding to three initial positions and vehicle angle as shown in Table 2. Destination point is same in all three cases which is at top centre having co-ordinates 100,200. Table 3 shows the distance travelled in each case in terms of points on graph in simulation window.

TABLE 2. SIMULATION RESULTS OF PATH TRACING BY LPA ALGORITHM

Simula-tion	Initial Position	Final Position	Vehicle Angle	Distance Travelled
I	10,20	100,200	30	248.66
II	180,60	100,200	60	244.88
III	180,100	100,200	270	200.90

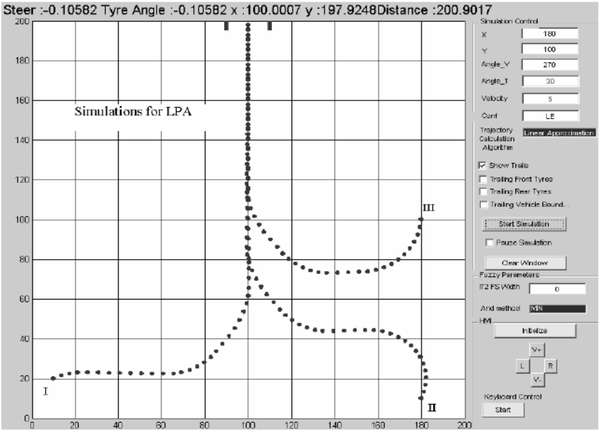

Fig 3: Path Traces by LPA Algorithm

From the simulation results, we contend that proposed algorithm has been successfully able to trace the path for nonholonomic vehicle between initial points to final point with varying initial vehicle angle.

V. CONCLUSION & FUTURE SCOPE

This work discussed design of a trajectory traversing technique, Linear Paths Approximation (LPA) and its integration with fuzzy logic based steering control system. Mathematical modeling has been done for path tracing between a source and target position, composed of small linear segments. Vast majority of the vehicles and robots are nonholonomic. It is always important to navigate autonomously a vehicle to obtain a precise final location & orientation in applications such as precision agriculture, horticulture, gardening, forestry, industrial works, social or civil works, space works, geo studies and a lot more applications. Proposed algorithm has been successfully tested in simulation environment using MATLAB.

Proposed work can further be integrated with overall vehicle navigation system on the hardware platform. Further the obstacles can be incorporated in the mathematical model so as to be utilized in the autonomous navigation.

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